Marriage Mathematics

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Abstract

The goal of this project was to apply group theory to the marriage laws of various closed societies. Given a set of rules that constitute the framework of the marriage system of a society, we explored all possible societal arrangements. These rules can be formulated in terms of an anthropologist's abstraction, a set of *marriage types.* Through application of group theory, we were able to arrive at conclusions about possible marriage systems under these rules. We tabulated all feasible societies with fewer than twelve marriage types. For each society we examined, we were able to evaluate the circumstances under which specific relatives could marry. We eventually used all of this information to help us interpret two societies which actually existed. The culmination of our project was application of our results to a reasonable, hypothetical society of our own.

I. Historical Background

In 1949, the anthropologist Claude Lévi-Strauss authored a book called *The Elementary Structures of Kinship*. It includes a chapter in which the mathematician André Weil examines how some of the marriage arrangements described by Lévi-Strauss can be systematically analyzed as algebraic structures whose elements are **marriage types**, conforming to certain rules.

Our project examined these structures, including some not subject to a commutativity rule imposed by Weil.

II. Statement of Problem

A. Rules

In this project, we studied marriage in a closed society by modeling systems with a finite set M of **marriage types**. The goal of this project was to determine every possible marriage system in a closed society that follows a certain set of rules. The systems examined here can be modeled by making the following assumptions:

Rule 1: *Every person has exactly one marriage type; male and female may marry if and only if they have the same marriage type.*

Rule 2: *One's marriage type is determined totally by one's parents' marriage type and one's sex.*

Rule 3: *Marriage types do not die out.*

Rule 4: *Marriage between siblings, and between parent and child, are not permitted.*

Rule 5: *Given a male and a female related through a common ancestor or a common descendant, whether or not they can marry each other depends only on the relationship.*

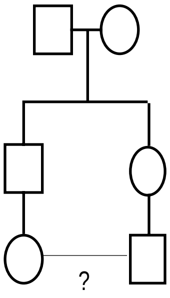
Rule 6: *Any two persons can have a common descendant, so that the society is not divided into segments that can not intermarry.*

We also set out to answer several questions about the conditions under which males and females in the society are allowed to marry. These questions are described in the next subsection.

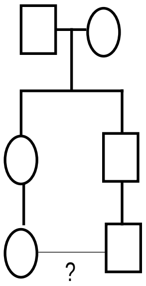
These rules and questions can be interpreted using mathematical proofs and family trees, as will be presented later in this report.

B. Questions

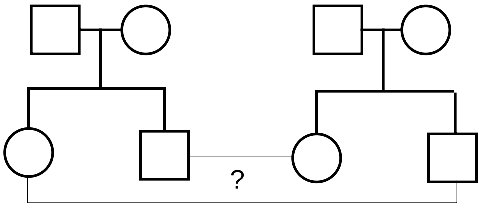
Family Trees for questions 1-3



Question 1: *Are true cross-cousins (i.e., a son of a daughter of x and a daughter of a son of x) allowed to marry?*



Question 2: *Are false cross-cousins (i.e., a son of a son of x and a daughter of a daughter of x) allowed to marry?*



Question 3: *Is a brother-sister pair allowed to marry a sister-brother pair?*

We intend to prove that affirmative answers to any two of these questions imply an affirmative answer to the third.

III. Mathematical Interpretation

A. Mathematical Preliminaries

We shall deal extensively with the set Perm M of all permutations of the finite set M. The **identity permutation** of M will be denoted by id.

Since there is no danger of confusion, **composition** of permutations will be denoted by juxtaposition. Thus, for all permutations φ, ψ of M and every $a \in$ M,

$$(\psi\varphi)(a) := \psi(\varphi(a)).$$

Consequently, the *k*th **iterate** of a permutation φ of M will be denoted by φ^k for all $k \in$ **N**.

The **inverse** of a permutation φ of M will be denoted by φ^-. We note that, since M is finite, φ^- is in fact an iterate of φ.

Every permutation φ of M partitions M into **orbits** such that $a,b \in$ M are in the same orbit of φ if and only if b $\in \varphi^k(a)$ for some $k \in$ **N.**

As usual, the permutation with just one non-singleton orbit traversed in the order *a, b, c,..., h* by iterating φ, is denoted by (*abc...h*).

B. Mathematical Analysis

In this section, we shall apply Rules 1 through 6 to develop propositions that are the core of our report.

Proposition 1: *There are mappings m and f from* M *to* M, *such that, for all a* $\in$ M, *m(a) and f(a) are the marriage types of a son and of a daughter, respectively, of parents with the shared marriage type a.*

Proof. This is an immediate consequence of Rules 1 and 2. n

Proposition 2: *m and f are permutations of* M, *i.e., m, f* $\in$ Perm M.

Proof. Because of Rule 3, every marriage type has to appear as a value of *m* and also of *f*; i.e., *m* and *f* must be surjective. Since M is finite, it follows from the Pigeonhole Principle that *m* and *f* are also injective, hence permutations of M.

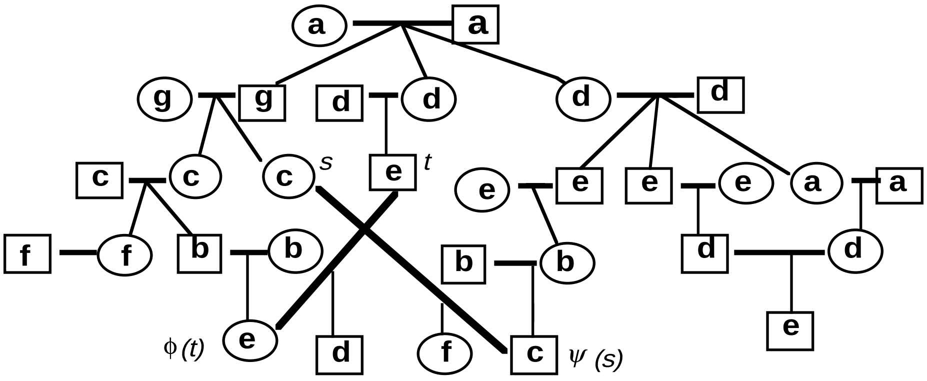
Proposition 3: id*, m, and f are distinct permutations.*

Proof. Following from Rule 4, a daughter may not have the same marriage type as her parents (otherwise she would be able to marry her father) and a son may not have the marriage type of his parents (otherwise he would be able to marry his mother); both children may not inherit the same marriage type (otherwise they would be able to marry each other).

At this point, we introduce the central concept of the paper. The group G is defined as the subgroup of Perm M obtained by composing strings of *f*, *m*, f^-, and m^-. From a remark in the previous subsection, we note that every member of G is actually the composite of a string of *f*'s and *m*'s. Rule 5 is key for the following statement.

Proposition 4: *If* $\varphi \in$ G *has a fixed point, then* $\varphi =$ id.

Proof. Let $\varphi \in$ G be given. Then we may choose a string of *f, m,* f^-, m^-, such that φ is its composite. This string corresponds to a certain relationship and φ maps the marriage type of a person to the marriage type of another person thus related to the first. We may assume that these two persons are of opposite sex; otherwise, we can achieve this by adding the pair f^-, *f* at the right end of the string, which leaves the composite φ unaltered. Rule 5 then allows us to conclude that either $\varphi(a) \neq a$ for all $a \in$ M (φ has no fixed point) or else $\varphi(a) = a$ for all $a \in$ M (φ is the identity).



φ=*fmfmf⁻m⁻* ψ = *mfmfm⁻f⁻*

Proposition 5: *All orbits of any given member of* G *have to be of the same size.*

Proof. Take a permutation $\psi \in$ G. Consider a least numerous orbit of ψ and call it *C*. Iterate ψ $|C|$ times. In the resulting iterate, all marriage types in *C* are fixed points. The permutation $\psi^{|C|}$ has fixed points and, by Proposition 4, must be the identity. This requires $|K|$ to divide $|C|$ for every orbit *K* of ψ; but since $|C| \leq |K|$ by the choice of *C*, it follows that $|K| = |C|$ for every orbit *K* of ψ.

Proposition 6: *For any given marriage types a, b* $\in$ M*, there is exactly one* $\psi \in$ G *such that* $\varphi(a) = b$.

Proof. Let $\varphi, \psi \in$ G satisfy $\varphi(a)=b=\psi(a)$. Then $(\psi^{-}\varphi)(a)=\Psi^{-}(b)=a$. From Proposition 4, it follows that $\psi^{-}\varphi$=id, and hence $\varphi=\psi$. Thus there is *at most one* $\varphi \in$ G such that $\varphi(a)=b$.

By Rule 6 there exists strings of *m* and *f* whose composites $\psi, \omega \in$ G satisfy $\psi(a)=\omega(b)$. Then $\omega^{-}\psi(a)=b$. Thus there is *at least one* $\varphi \in$ G such that $\varphi(a)=b$.

This essentially means that each element of M must be able to be mapped by a member of G to every other element in M. If Rule 6 were to fail, the population would break up into distinct segments. The separation would be permanent and the segments would be genetically isolated. Rules 1 to 6 would then apply to each of these segments separately.

Proposition 7: |G|=|M|.

Proof. Choose $a \in$ M. Then associating $\varphi \in$ G with $\varphi(a) \in$ M establishes, by Proposition 6, a one-to-one correspondence between G and M.

We will now interpret in mathematical terms the three questions posed before. For Question 1, if the first generation is assigned a marriage type *t*, then the marriage type of the son's daughter would be *fm*(*t*), while the marriage type of the daughter's son would be *mf*(*t*). Therefore, true cross-cousins are allowed to marry if and only if *fm=mf*. This condition is equivalent to the commutativity of the group G. In Question 2, the daughter's daughter would have a marriage type of $ff(t)$, and the son's son type would then be $mm(t)$. False cross-cousins are allowed to marry if and only if $mm=ff$. For Question 3, if t is the marriage type of a given couple, the marriage types of the husband's sister and the wife's brother are $fm^{-}(t)$ and $mf^{-}(t)$, respectively. Thus, a brother-sister pair may marry a sister brother pair if and only if $fm^{-} = mf^{-}$.

Proposition 8: *Affirmative answers to any two of Questions 1, 2, and 3 imply an affirmative answer to the remaining question.*

Proof. 1. Assume that $mm=ff$ and $mf=fm$ (G is commutative, so $fm^{-}=m^{-}f$). Then $mf^{-}=mmm^{-}f^{-}=ffm^{-}f^{-}=fm^{-}ff^{-}=fm^{-}$.

Thus, affirmative answers to Questions 1 and 2 imply an affirmative answer to Question 3.

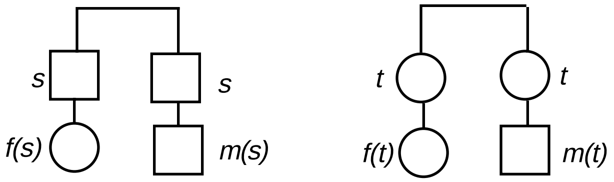
2. Assume that $mm=ff$ and $mf^{-}=fm^{-}$. Then $mf=mf^{-}ff=fm^{-}mm=fm$.

Thus, affirmative answers to Questions 2 and 3 imply an affirmative answer to Question 1.

3. Assume that $mf=fm$ (G is commutative) and $mf^{-}=fm^{-}$. Then $mm=mf^{-}fm=fm^{-}mf=ff$.

Thus, affirmative answers to Questions 1 and 3 imply an affirmative answer to Question 2.

Proposition 9: *Parallel cousins (i.e., a woman and a man whose fathers are brothers or whose mothers are sisters) can not marry.*



$f(s) \neq m(s)$ $f(t) \neq m(t)$

Proof. From Rule 2, the brothers (or sisters) will have the same marriage type s. The marriage type of a daughter of a brother would then be $f(s)$ and the marriage type of a son of a brother would be $m(s)$. These marriage types are the same as those of a sister-brother pair and so, by Rule 4, these two cousins can not marry.

III. Groups

A. Preliminaries

In this section, we intend to present all possible systems satisfying the rules with up to eleven marriage types. Note that Propositions 3 and 6 imply that there are at least three marriage types, and Proposition 7 asserts that $|G|=|M|$. We consider separately the cases in which $|M|$ is an odd prime.

Two systems are said to be **isomorphic** if one may be obtained from the other by permuting the *names* of the marriage types. In presenting the systems in this section, only non-isomorphic systems will be recorded.

The concept of duality is useful when presenting the different possible societies for a given group. Two systems are said to be **dual** to each other if one is transformed into the other when the sexes are reversed. The dual of a system has the same group structure of G as the original system, but the generators *m* and *f* are interchanged. For example, if for a certain system the relation $m = f^3$ holds, then for its dual system the relation $f = m^3$ would hold. Some systems are **self-dual**, meaning that they are their own duals. In particular, this is the case when $m = f^{-}$, which may occur when the group G is a cyclic group. In self-dual systems, there is a kind of "equality" between males and females, in that they are symmetric with respect to the marriage laws. (This does not mean they are equal in other respects, though!)

B. Primes

If |M| = *p*, where *p* is an odd prime, then by Proposition 5, each permutation in G must be either the identity or a cycle of size *p*. Every cycle of size *p* generates the entire cyclic group of order *p*; therefore G must be the cyclic group of order *p*. Every non-identity element of G is a cycle of size *p*, and thus generates the entire group. Thus, *m* can be assigned to be any non-identity element α in G. G then consists of the iterates of α. The assignment of *f* may not be to the identity nor to *m,* according to Proposition 3. Hence, *f* must be assigned to one of the remaining (*p* - 2) iterates of α (i.e. { α^2, α^3, ... α^{p-1}}). No two of the resulting (*p* - 2) systems are isomorphic. In presenting the systems, it may be desirable to replace some of the systems just described by isomorphic ones obtained by choosing a different generator for *m*, in order to bring out the duality between some pairs of systems.

To illustrate these general results, consider the case of five marriage types. The only possible group G is the cyclic group of order five, shown below, with an arbitrary but specific choice of α, for M:={a,b,c,d,e}.

id	id
(abcde)	α^1
(acebd)	α^2
(adbec)	α^3
(aedcb)	$\alpha^4 = \alpha^{-}$

There are three possible non-isomorphic choices of *m* and *f* from this group, where the two systems in the first row are dual to each other.

systems		1	2	3
$m = \alpha \;\; f = \alpha^2$	$f = \alpha \;\; m = \alpha^2$	yes	no	no
$m = \alpha \;\; f = \alpha^4 = \alpha^{-}$		yes	no	no

Note that $f = \alpha \;\; m = \alpha^2$ is isomorphic to $m = \alpha \;\; f = \alpha^3$.

Since a cyclic group is always commutative, Question 1 always has an affirmative answer for cyclic groups, and consequently when |M| is an odd prime. Question 3 has an affirmative answer only if G has an element of order two: since $mf^{-} = fm^{-}$, this implies $(mf^{-})^2$ = id; thus the answer is negative for all cases where |M| is an odd prime. According to Proposition 8, because Question 1 always has an affirmative answer and Question 3 always has a negative answer for all primes, Question 2 always has a negative answer for all primes.

C. Composites

In order to illustrate the process that we followed for all the systems we examined, we will walk step by step through the systems with four marriage types.

We consider M: ={a,b,c,d}

Since M has four members, every member of G other than the identity will have, by Proposition 5, one orbit with four members (i.e., all of M) or two orbits of two members each.

Suppose first that some member, α, of G has M as an orbit. Then G={id, $\alpha, \alpha^2, \alpha^3$} is a cyclic group. We must determine which members of G are *m* and *f.* Up to permutation of the names of the marriage types, the only choices conforming to Proposition 3 are:

$$m=\alpha,\ f=\alpha^2 \qquad m=\alpha,\ f=\alpha^3 \qquad f=\alpha,\ m=\alpha^2$$

Note that the first and third choices are dual to each other.

The groups of prime order were dealt with in the previous subsection, hence here we present only the systems with four, six, eight, nine, and ten marriage types.

For each group, the first table gives the permutations with their corresponding notations. The second lists all the possible societies in dual pairs and answers the three questions that were posed earlier.

Four Marriage Types

cyclic group — an element of order four and its iterates

id	id
(a b c d)	α
(a c) (b d)	α^2
(a d c b)	$\alpha^3=\alpha^-$

systems		1	2	3
$m=\alpha\ \ f=\alpha^2$	$f=\alpha\ \ m=\alpha^2$	yes	no	no
$m=\alpha\ \ f=\alpha^3=\alpha^-$		yes	yes	yes

commutative non-cyclic group — all elements of order two

id	id
(a b) (c d)	α
(a c) (b d)	β
(a d) (b c)	$\alpha\beta=\beta\alpha$

system	1	2	3
$m = \alpha \quad f = \beta$	yes	yes	yes

Six Marriage Types

cyclic group — an element of order six and its iterates

id	id
(a b c d e g)	α
(a c e) (b d g)	α^2
(a d) (b e) (c g)	α^3
(a e c) (b g d)	α^4
(a g e d c b)	$\alpha^5 = \alpha^-$

systems		1	2	3
$m = \alpha \quad f = \alpha^2$	$f = \alpha \quad m = \alpha^2$	yes	no	no
$m = \alpha \quad f = \alpha^3$	$f = \alpha \quad m = \alpha^3$	yes	no	no
$m = \alpha \quad f = \alpha^4$	$f = \alpha \quad m = \alpha^4$	yes	yes	yes
$m = \alpha \quad f = \alpha^5 = \alpha^-$		yes	no	no
$m = \alpha^2 \quad f = \alpha^3$	$f = \alpha^2 \quad m = \alpha^3$	yes	no	no

non-commutative group

id	id
(a b c) (d e g)	α
(a c b) (d g e)	α^-
(a d) (b g) (c e)	β
(a e) (b d) (c g)	$\alpha \beta = \beta \alpha^2$
(a g) (b e) (c d)	$\beta \alpha = \beta \alpha$

systems		1	2	3
$m = \alpha \quad f = \beta$	$f = \alpha \quad m = \beta$	no	no	yes
$m = \beta \quad f = \alpha \beta$		no	yes	no

Eight Marriage Types

cyclic group — an element of order eight and its iterates

id	id
(a b c d e g h i)	α
(a c e h) (b d g i)	α^2
(a d h b e i c g)	α^3
(a e) (b g) (c h) (d i)	α^4
(a g c i e b h d)	α^5
(a h e c) (b i g d)	α^6
(a i h g e d c b)	$\alpha^7 = \alpha^-$

systems		1	2	3
$m=\alpha$ $f=\alpha^2$	$f=\alpha$ $m=\alpha^2$	yes	no	no
$m=\alpha$ $f=\alpha^3$		yes	no	no
$m=\alpha$ $f=\alpha^4$	$f=\alpha$ $m=\alpha^4$	yes	no	no
$m=\alpha$ $f=\alpha^5$		yes	yes	yes
$m=\alpha$ $f=\alpha^6$	$f=\alpha$ $m=\alpha^6$	yes	no	no
$m=\alpha$ $f=\alpha^7=\alpha^-$		yes	no	no

commutative non-cyclic group — generated by an element of order four and an element of order two

id	id
(a b c d) (e g h i)	α
(a c) (b d) (e h) (g i)	α^2
(a d c b) (e i h g)	$\alpha^3 = \alpha^-$
(a e) (b g) (c h) (d i)	$\beta = \beta^-$
(a g c i) (b h d e)	$\beta\ \alpha$
(a h) (b i) (c e) (d g)	$\beta\ \alpha^2$
(a i c g) (b e d h)	$\beta\ \alpha^3$

systems		1	2	3
$m=\alpha$ $f=\beta$	$f=\alpha$ $m=\beta$	yes	no	no
$m=\alpha$ $f=\beta\ \alpha$		yes	yes	yes

first non-commutative group

id	id
(a b c d) (e g h i)	α
(a c) (b d) (e h) (g i)	α^2
(a d c b) (e i h g)	$\alpha^3 = \alpha^-$
(a i) (b h) (c g) (d e)	$\beta = \beta^-$
(a h) (b g) (c e) (d i)	$\beta\,\alpha = \alpha^3 \beta$
(a g) (b e) (c i) (d h)	$\beta\,\alpha^2 = \alpha^2 \beta$
(a e) (b i) (c h) (d g)	$\beta\,\alpha^3 = \alpha\,\beta$

systems		1	2	3
$m = \alpha\ \ f = \beta$	$f = \alpha\ \ m = \beta$	no	no	yes
$m = \beta\ \ f = \beta\,\alpha$		no	yes	no

second non-commutative group:

id	id
(a b c d) (e g h i)	α
(a e c h) (b i d g)	β
(a i c g) (b h d e)	$\beta\,\alpha = \beta^- \alpha$
(ad c b) (e i h g)	α^-
(a h c e) (b g d i)	β^-
(a g c i) (b e d h)	$(\beta\,\alpha)^- = \alpha^- \beta^- = \alpha\,\beta$
(a c) (b d) (e h) (g i)	$\alpha^2 = \beta^2$

system	1	2	3
$m = \alpha\ \ f = \beta$	no	yes	no

Nine Marriage Groups

cyclic group — an element of order nine and its iterates

id	id
(a b c d e g h i j)	α
(a c e h j b d g i)	α^2
(a d h) (b e i) (c g j)	α^3
(a e j d i c h b g)	α^4
(a g b h c i d j e)	α^5
(a h d) (b i e) (c g j)	α^6
(a i g d b j h e c)	α^7
(a j i h g e d c b)	$\alpha^8 = \alpha^-$

systems		1	2	3
$m = \alpha \;\; f = \alpha^2$		yes	no	no
$m = \alpha \;\; f = \alpha^3$	$f = \alpha \;\; m = \alpha^3$	yes	no	no
$m = \alpha \;\; f = \alpha^4$		yes	no	no
$m = \alpha \;\; f = \alpha^6$	$f = \alpha \;\; m = \alpha^6$	yes	no	no
$m = \alpha \;\; f = \alpha^8 = \alpha^-$		yes	no	no

commutative group — generated by two elements of order three

id	id
(a b c) (d e g) (h i j)	α
(a c b) (d g e) (h j i)	$\alpha^2 = \alpha^-$
(a d h) (b e i) (c g j)	β
(a e j) (b g h) (c d i)	$\beta\,\alpha$
(a g i) (b d j) (c e h)	$\beta\,\alpha^2 = \beta\,\alpha^-$
(a h d) (b i e) (c j g)	$\beta^2 = \beta^-$
(a i g) (b j d) (c h e)	$\beta^2\,\alpha$
(a j e) (b h g) (c i d)	$\beta^2\,\alpha^2$

systems	1	2	3
$m = \alpha \;\; f = \beta$	yes	no	no

Ten Marriage Groups

cyclic group — an element of order ten and its iterate

id	id
(a b c d e g h i j k)	α
(a c e h j) (b d g i k)	α^2
(a d h k c g j b e i)	α^3
(a e j c h) (b g k d i)	α^4
(ag) (b h) (c i) (d j) (e k)	α^5
(a h c j e) (b i d k g)	α^6
(a i e b j g c k h d)	α^7
(a j h e c) (b k i g d)	α^8
(a k j i h g e d c b)	$\alpha^9 = \alpha^-$

systems		1	2	3
$m=\alpha\ f=\alpha^2$	$f=\alpha\ m=\alpha^2$	yes	no	no
$m=\alpha\ f=\alpha^3$	$f=\alpha\ m=\alpha^3$	yes	no	no
$m=\alpha\ f=\alpha^4$	$f=\alpha\ m=\alpha^4$	yes	no	no
$m=\alpha\ f=\alpha^5$	$f=\alpha\ m=\alpha^5$	yes	no	no
$m=\alpha\ \mathrm{i}=\alpha^6$	$f=\alpha\ m=\alpha^6$	yes	yes	yes
$m=\alpha\ f=\alpha^8$	$f=\alpha\ m=\alpha^8$	yes	no	no
$m=\alpha\ f=\alpha^9=\alpha^-$		yes	no	no

non-commutative group — generated by an element of order five and an element of order two

id	id
(a b c d e) (g h i j k)	α
(a c e b d) (g i k h j)	α^2
(a d b e c) (g j h k i)	α^3
(a e d c b) (g k j i h)	α^4
(a g) (b k) (c j) (d i) (e h)	β
(a k) (b j) (c i) (d h) (e g)	$\beta\ \alpha$
(a j) (b i) (c h) (d g) (e k)	$\beta\ \alpha^2$
(a i) (b h) (c g) (d k) (e j)	$\beta\ \alpha^3$
(a h) (b g) (c k) (d j) (e i)	$\beta\ \alpha^4$

systems		1	2	3
$m=\alpha\ \ f=\beta$	$f=\alpha\ \ m=\beta$	no	no	yes
$m=\beta\ \ f=\beta\ \alpha$		no	yes	no

IV. Implementation

A. Actual Systems

The Kariera and Aranda systems were actual Aboriginal societies in Australia that illustrate the mathematical models just developed.

The Kariera System

In the Kariera system, there are two matriclans and two patrilocales. The combinations of the clans and locales have the names recorded in the table on the folllowoing page:

	patrilocales	
matriclans	Burung	Palyeri
	Karimera	Banaka

Two people can marry if and only if they are of different clans and different locales, so there are four possible marriages, or marriage types.

male	female	marriage type
Burung	Banaka	a
Banaka	Burung	b
Karimera	Palyeri	c
Palyeri	Karimera	d

The Kariera system is matrilineal, meaning that a person inherits his or her mother's clan, and patrilocal, meaning that a person inherits his or her father's locale. For example, the child of a Burung father and a Banaka mother (marriage type a) would be Karimera. If the child were then male, he would have marriage type c; if female, she would have marriage type d. Continuing in this manner, it is found that the permutations *m* and *f* are

$$m=(a\,c)\,(b\,d) \quad f=(a\,d)\,(b\,c)$$

The group generated by these permutations is the (commutative) non-cyclic group for four marriage types.

The Aranda System

In the Aranda system, there are four main clans, each consisting of two subclans. The eight subclans will be denoted by the symbols A, a, B, b, C, c, D, and d. The possible marriages and the subclans of the children are given in the following table, which also lists the values of *m* and *f*.

father mother	child	*m*	*f*
A - B	d	d - c	c - d
a - b	D	D - C	C - D
B - A	C	C - D	D - C
b - a	c	c - d	d - c
C - D	B	B - A	A - B
c - d	b	b - a	a - b
D - C	a	a - b	b - a
d - c	A	A - B	B - A

Permutation *m* is of order 2 and permutation *f* is of order 4. Together they generate the first non-commutative group for eight marriage types given in Section III. In this example, the assignments for *m* and *f* are (after suitably permuting the names of the marriage types)

$$f=\alpha \qquad m=\beta .$$

B. The PeGaSuS system

The final endeavor of our team project was to use the background information and knowledge we acquired to create our own fictitious society, one that could have possibly existed. The society we formulated, PeGaSuS, consists of six marriage types. Each marriage type is described by two components. The first component is a symbol which represents the person's occupation. There are three **categories** of occupations: **knife** which consists of occupations such as physicians and hunters; **rock** which represents carpenters and masons; and **papyrus** which includes lawyers and secretaries. A person's category of occupation dictates dominance with respect to marriage. This **dominance** is traditionally recorded through this proverb: *rock smashes knife, knife cuts papyrus, papyrus wraps rock.* For each marriage, the female's occupation must dominate the male's. All children then inherit the mother's category of occupation.

The second component of the marriage type description is a **generation label.** There are two generation labels, x and y, and two people who wish to marry must bear the same generation label. Their children will then receive the opposite generation label. So for instance, if the grandparents of a family are of generation x, then the children will be of generation y, and consequently the grandchildren will be of generation x.

Since remote antiquity and until July of 1984, this system was in force, and may be modeled as follows:

	husband	wife	child
a	Kx	Sx	Sy
b	Ky	Sy	Sx
c	Px	Kx	Ky
d	Py	Ky	Kx
e	Sx	Px	Py
g	Sy	Py	Px

So that m = (a g c b e d) and f = (ab) (cd) (eg).

This is described by a commutative group G, since $mf = fm$; and so true cross cousins are able to marry, but because $mm \neq ff$ and $mf^- \neq fm^-$, false cross cousins cannot marry, nor can brother-sister pairs marry. Specifically, m is of order six, so it generates the cyclic group for six marriage types. This system corresponds to the cyclic group for six marriage types given in Section III, with the assignments for m and f being $m = \alpha$, $f = \alpha^3$.

In July of 1984, the men of the PeGaSuS society revolted and demanded that theirs be the dominant occupation in marriages. The society came to a compromise, and it was decided that a single change would be made; thenceforth, all marriages of generation y would require the male to have the dominant occupation. This change in the dominance rules altered the entire system. It would now be modeled as follows:

	husband	wife	child
a	Kx	Sx	Sy
b	Ky	Py	Px
c	Px	Kx	Ky
d	Py	Sy	Sx
e	Sx	Px	Py
g	Sy	Ky	Kx

So that m = (ag) (bc) (de) and f = (ad) (be) (cg)

This is an example of the non-commutative group for six marriage types. Since $mf \neq fm$ and $mf^- \neq fm^-$, true cross cousins cannot marry nor can brother-sister pairs marry. Because $mm = ff$, false cross cousins can marry. The society thus became non-commutative. This system corresponds to the non-commutative group for six marriage types given in Section III, with the assignments for m and f being $m = \beta$, $f = \alpha\beta$.

VII. Conclusion

We considered societies obeying certain kinds of marriage rules, interpreted them in mathematical terms, and derived various propositions of mathematical and anthropological interest. We then used these propositions to analyze all such societies with fewer than twelve marriage types. These mathematical models were shown to be implemented by the actual Australian societies of the Kariera and Aranda, and with a reasonable, if fictitious, society of our own, PeGaSuS, in two different modes. This topic is a fascinating example of group theory having an application in a field where mathematics is not greatly used, the social sciences.

VIII. References

Benjamin Baumslag and Bruce Chandler, *Theory and Problems of Group Theory*. (McGraw-Hill Book Company, New York, 1968).

Claude Lévi-Strauss, *The Elementary Structure of Kinship*, (Beacon Press, Boston, 1969).

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